

A Deep Neural Network Based Quasi-Linear Kernel for Support Vector Machines

Weite LI and Jinglu HU

Graduate School of Information Production and Systems, WASEDA University

1. SVM with a Quasi-Linear Kernel

$$f_p(x) = \sum_{j=1}^M (\Omega_j^T x + b_j) R_j(x) + b$$

$$\Phi(x) = [R_1(x), x^T R_1(x), \dots, R_M(x), x^T R_M(x)]^T$$

$$\Theta = [b_1, \Omega_1^T, \dots, b_M, \Omega_M^T]^T$$

$$f_p(x) = \Theta^T \Phi(x) + b$$

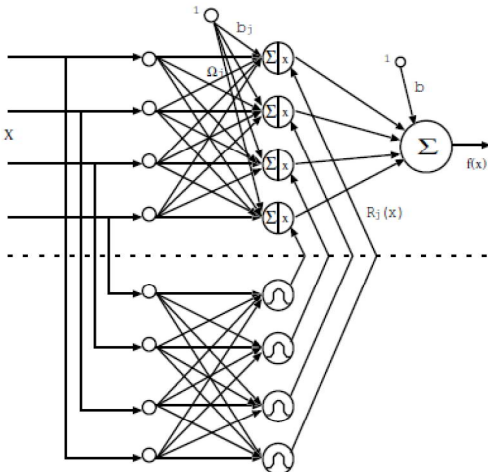
$$\max_{\alpha} \mathcal{J}_D(\alpha) = -\frac{1}{2} \sum_{k,l=1}^N y_k y_l K(x_k, x_l) \alpha_k \alpha_l + \sum_{k=1}^N \alpha_k$$

$$s.t. \begin{cases} \sum_{k=1}^N \alpha_k y_k = 0 \\ 0 < \alpha_k < c, k = 1, \dots, N \end{cases}$$

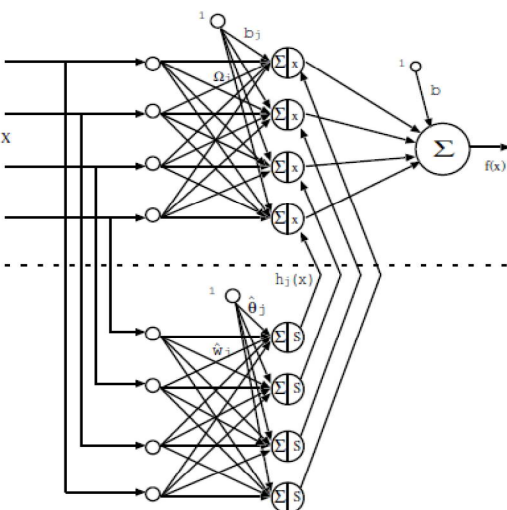
$$K(x_k, x_l) = \Phi^T(x_k) \Phi(x_l) = (1 + x_k^T x_l) \sum_{j=1}^M R_j(x_k) R_j(x_l)$$

2. A Network View

- When $R_j(x) = \exp\left(-\|x - \mu_j\|_F^2 / \lambda \sigma_j^2\right)$, it provides a flexible and adjustable kernel, filling the gap between linear and nonlinear kernels.

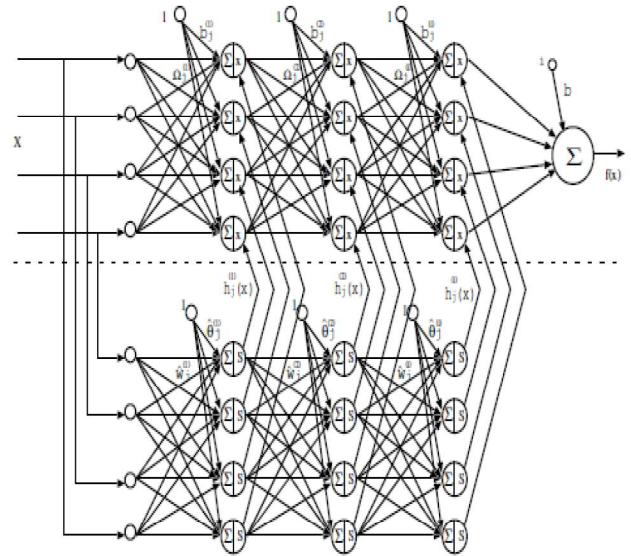


- When $R_j(x) = \begin{cases} 1, & \hat{w}_j^T x + \hat{\theta}_j > 0 \\ 0, & \hat{w}_j^T x + \hat{\theta}_j \leq 0 \end{cases}$, it mimics the functionality of a rectified network with single hidden unit, whose multilayer versions are widely used.

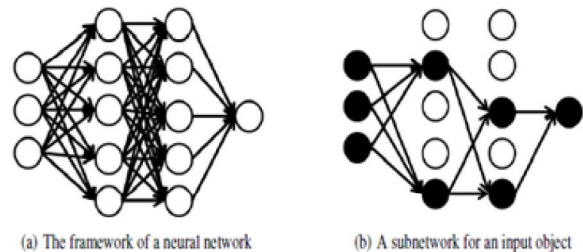


3. An Extension to Multilayer Version

- When multiple layers stacked together, a **deep quasi-linear kernel** can be built upon it.



- It uses the subnetworks of a **pre-trained deep neural network** to build a kernel for SVM. A subnetwork looks like the followings, which only comprises a subset of parameters.

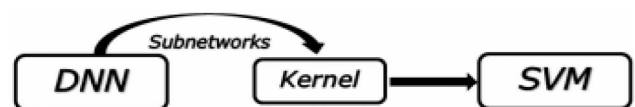


- It leads to a **more advanced transfer learning scheme**, which makes the tuning of deep neural networks also available for small data sets without the sacrifice of generalization ability.



(a) The transfer of features

Towards a more advanced transfer learning scheme



(b) The transfer of subnetworks